

rpm was assumed to be suspended using four equally spaced magnetic actuator stations. The nominal rim center was arbitrarily located on the plate.

Choosing  $K_p$  and  $K_r$  to be diagonal matrices with equal entries ( $k_p$  and  $k_r$ , respectively), root loci were generated by varying  $k_p$  and  $k_r$ . Figure 2 shows a typical root locus (first seven modes) for  $k_p = 146$  N/m and  $k_r$  varied from 0 to 19,919 N-s/m. All closed-loop eigenvalues were inside the left half-plane for  $k_p > 0$  and  $k_r > 0$ . Thus, although the sufficient condition for mere stability was satisfied, the system is asymptotically stable. The closed-loop damping ratios (for nominal design) for modes 1, 2, 3, 5, and 7 were 0.13, 0.13, 0.44, 0.2, and 0.1, respectively. Damping on modes 4 and 6 does not improve much because the actuator location is not favorable for these modes. In practice, actuator locations must be chosen so as to get the maximum effect on the most important modes. Although only the first seven modes are shown, all other modes also behave in the same manner.

In order to demonstrate the importance of angular momentum  $H$ , gains  $k_p$  and  $k_r$  were fixed at  $k_p = 146$  N/m and  $k_r = 5615$  N-s/m, and  $H$  was varied from 0 to 4 times its nominal value. At  $H=0$ , there is very little damping (only because of the small mass of the rim). The damping improved as  $H$  increased, and exhibited a turnaround for some of the modes.

### Conclusions

This paper proposes the use of an annular momentum control device (AMCD) for enhancing the modal damping of large space structures (LSS's) during fine pointing missions. It has been proved that an AMCD cannot destabilize the LSS. The control concept presented requires no knowledge of the

LSS model, and is stable regardless of the number of modes in the model. Numerical results obtained for a large, thin, completely free undamped aluminum plate indicate that an asymptotically stable closed-loop system can be obtained with satisfactory modal damping ratios. This damping enhancement system, when used in conjunction with a primary controller that controls the LSS rigid-body modes and selected structural modes, has significant potential.

### Appendix

**Lemma:** Let  $P$  be an  $n \times m$  matrix ( $m \leq n$ ) of rank  $m$ . If  $\lambda$  is an eigenvalue of  $P(P^T P)^{-1} P^T$ , then  $\lambda = 0$  or  $\lambda = 1$ .

**Proof:** Consider the eigenvalue equation

$$P(P^T P)^{-1} P^T x = \lambda x \quad (A1)$$

Premultiplying by  $P^T$ ,

$$P^T x = \lambda P^T x \quad (A2)$$

Therefore, if  $P^T x \neq 0$ , then  $\lambda = 1$ . If  $P^T x = 0$  and  $x \neq 0$ , then from Eq. (A1),  $\lambda = 0$ .

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## Technical Comments

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### Comment on "Method of Optimizing the Update Intervals in Hybrid Navigation Systems"

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LEONDES, Phillis, and Chin<sup>1</sup> have calculated a "lower bound" on the measurement update interval for a hybrid navigation Kalman filter. The proposed lower bound is the smallest update interval for which the innovation sequence  $\delta \tilde{z}(k) = \delta z(k) - H(k) \delta \hat{x}(k)$  is not significantly time-correlated. The authors state that, if the update interval is smaller than this lower bound, the process model will be invalid because the assumption of white measurement noise will be unjustified. This would be correct if the measurement

noise were assumed to be white. However, this whiteness assumption is not necessary (nor is it physical, as the authors point out), and in fact Kalman filters may be synthesized when state noise and measurement noise are assumed to be colored.<sup>2</sup> For a filter so designed, time-correlation of the sequence  $\delta \tilde{z}(k)$  is expected and is not indicative of invalidity of the model. If, on the other hand, a Kalman filter has been designed based on an assumption of white measurement noise, improved performance might be obtained by redesigning the filter to model the colored noise and then decreasing the update interval to the smallest computationally feasible interval.

The authors also err in stating that "Sampling at time points spaced by  $\Delta t$  is analogous to passing the continuous noise through a bandpass filter of bandwidth  $B = 2\pi/\Delta t$ ." If the sampling is ideal (i.e., impulsive), then there is no attenuation of high-frequency components, and aliasing will occur. More realistically, if the sampling pulse width is small but nonzero, the sampling process will pass (and, in fact, generate) components at frequencies much higher than the sampling frequency.

### References

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